A mathematical analysis of the 4 × 100 m relay

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In this study, we examined aspects of the 4 × 100 m relay that are amenable to mathematical analysis. We looked at factors that affect the time required to complete the relay, focusing on the performance of elite male athletes. Factors over which the individual athletes, and the team coach, can exercise some control are: the starting positions of the runners on legs 2, 3 and 4, the positions at which baton exchanges occur, the free distances at the baton exchanges and the running order of the athletes. The lane draw is shown to have an important influence on the relay time, although it is outside the control of the team coach. Teams drawn in the outside lanes benefit from the inverse relation between bend radius of curvature and running speed. For teams composed of athletes with different times over 100 m, we show that the fastest relay times are achieved with the fastest athlete taking the first leg, with the slowest two runners allocated to the final two legs.

Keywords: athletics, running, sprinting.

Introduction

The aim of this study was to examine aspects of the 4 × 100 m relay that are amenable to mathematical analysis. The basic building block of the 4 × 100 m relay is the 100 m sprint, a track event that has been subject to substantial mathematical analysis (see, for example, Ward-Smith, 1999). Perhaps surprisingly, little has been published on the 4 × 100 m relay. We sought to identify the factors that are of greatest importance in determining the time taken to complete a 4 × 100 m relay, focusing on the performance of elite male athletes.

The geometry of the race track used in running is defined by the International Amateur Athletic Federation (IAAF, 1999). The rules pertaining to the 4 × 100 m relay are as follows. Between the starting and finishing lines, the distance of 400 m is divided into four equal lengths of 100 m, defined by scratch lines at 100 m, 200 m and 300 m. At the end of the first three legs, takeover zones are defined, in which the baton is exchanged between the incoming and outgoing runners. The takeover zones are 20 m long and are symmetrically placed about each scratch line. Members of the relay team, other than the first runner, can start running not more than 10 m outside the takeover zone. Due

allowance is made for bend curvature in the layout of the starting lines, which are staggered across the eight lanes of the track. These rules effectively determine the distances run by the runners on each of the four legs.

For a 4 × 100 m relay team to be successful, consideration must be given to selecting the right runners and putting them in the right order. The relative skills of the athletes in starting and running a bend, and their preferred hand for giving and receiving a baton, are all factors in that success. The lane draw is another factor, although in the finals seeding reduces some of the disparities from this effect. Most of all, however, the athletes must ensure the greatest possible speed of the baton, not only on each leg of the relay, but also during each of the three exchanges, while striving to avoid dropping the baton or disqualification by overrunning the exchange zone.

Both the experimental measurements of Moravec *et al.* (1988) and the mathematical model of Ward-Smith and Radford (2000a) show that, during sprinting over 100 m on a straight track, an elite male athlete reaches his peak running speed some 60 m after leaving the starting blocks. The initial acceleration is rapid; after 20 m, the running speed is some 89% of the peak value. After 30 m, the running speed is about 94% of the peak value, so that the next 30 m account for a further increase in speed of as little as 6%. Beyond peak running speed, a small but consistent decline occurs until the

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finishing line is reached. These considerations have an important bearing on the conduct of the 4×100 m relay. In the relay, the elite sprinters running legs 2, 3 and 4 have the opportunity to accelerate to near full sprinting speed before taking over the baton. Hence, the athletes on these last three legs need to use most of the available 30 m for acceleration from rest, providing that in extending the distance over which they run they do not suffer a disproportionate loss of speed in the later stages. As we will show later (Table 2), the speed gained during the acceleration phase more than outweighs the gradual decline in speed experienced over the later stages. Consequently, it soon becomes evident that, for optimum team performance in the relay, the sprinters on the second and third legs, between starting from rest and handing over the baton, are required to cover a distance of well over 100 m, possibly approaching 130 m overall. The athlete on the first leg starts from rest on the starting line and covers some 110 m, while the runner on the fourth leg covers a distance of up to 120 m before crossing the finishing line.

The speed a runner can achieve running round a curved section of track is less than the speed that can be attained along a straight section (Greene, 1985), even though in both cases the athlete runs with maximum effort. To maintain the curved path, the runner must generate a lateral, horizontal component of ground reaction force to create the centipetal acceleration required and must lean into the bend to balance the moment this force creates. The need to develop this additional force component when running round a bend leads inevitably to a reduction in running speed, the magnitude of which depends in part on the straight-line speed of the athlete and in part on the radius of curvature of the bend. The effect of track curvature on running speed has been investigated comprehensively by Greene (1985).

The aim of the present study was to explore the effects on 4×100 m relay performance of changing various factors amenable to mathematical analysis. We examined the relationship between the overall relay time and the component 100 m times, and compared computed times with measured times on the track. As track curvature has an important influence on the outcome of the 4×100 m relay, we assessed the effect of this factor on running performance across the eight lanes of the track. We also looked at the effect on relay time of changing the positions from which the athletes on the final three legs start running and at which the baton exchanges occur. The effect of free distance – that is, the distance separating the two runners at the baton exchanges – was also investigated. Finally, we examined the effect on relay time of altering the running order of athletes when the relay team is composed of runners with different performances over 100 m. Put another

way, we sought to answer the question: What is the optimum order in which to deploy athletes with different 100 m performances?

Methods

Basic considerations

This study of the men's 4×100 m relay was built from several component parts. Reliable information on the performance of elite male sprinters running 100 m on straight track formed the starting point. The extension of the performance data up to a distance of 130 m was then considered. This was followed by the transformation of the straight-line data to provide a running profile relevant to movement over a curved track. Finally, the performances of the individual runners over the four legs were combined to yield the overall relay performance.

For the straight-line performance data over 100 m, we used the mathematical model of Ward-Smith and Radford (2000a). Besides predicting the performance of sprinters in still air conditions, the method is capable of accounting for the effects of tail or head winds. A particular feature of this method is its ability to predict not just overall running time, but also the intermediate times at intervals along the track over the course of the entire 100 m. The method has been used to model the average performance of the competitors in the 100 m final of the 1987 World Championships (Ward-Smith and Radford, 2000a), which was completed in an average time of 10.11 s and run with an assisting wind of 0.95 m·s^{-1}. To two decimal places, when corrected to zero wind speed (Ward-Smith and Radford, 2000b), the method yielded a time of 10.16 s over 100 m, made up of an average reaction time of 0.18 s plus a running time of 9.98 s. As has been shown already, some members of a relay team may be required to run a distance approaching 130 m during the relay. Hence, in this study, the calculations of Ward-Smith and Radford (2000a), which are based on the analysis of the energy conversion processes that take place during running and assume running with maximum effort throughout, have been extended beyond 100 m up to a distance of 130 m, using the same biophysical parameters as used in the original paper. Detailed experimental data are not available to validate the analytical data between 100 and 130 m. Thus, although the running profile has been fully validated over the first 100 m, from 100 to 130 m the profile must be regarded as an extrapolation. In the interests of clarity and to highlight the effects of various factors, we felt it to be an advantage to base the present calculations of relay performance on a simple round-figure performance over 100 m. For that reason, a repre-

sentative performance under zero wind conditions of 10.00 s over 100 m was used, made up of a reaction time of 0.18 s and a running time of 9.82 s.

Because adjustments to the running profile of less than ±2% are involved, a simple linear transformation was applied to the results of Ward-Smith and Radford (2000a) to derive the representative running performance on which all the calculations in this study were based. The starting point is the relation describing the average performance of the finalists in the 1987 World Championships, corrected to zero wind speed. This can be written as

$$t_W(x) = t_{W0}(x) + t_d \tag{1}$$

where $t_W(x)$ is the time to reach a distance x from the starting line calculated according to the theory of Ward-Smith and Radford (2000a), $t_{W0}(x)$ is the time to run a distance x for zero reaction time delay and t_d is the reaction delay time.

We define $t_T(x)$ as the time to reach a distance x from the starting line during the course of a run that takes time T to cover 100 m. The time $t_T(x)$ is made up of two components, a reaction time delay t_d and $t_{T0}(x)$, the time to run a distance x for zero reaction time. Hence

$$t_T(x) = t_{T0}(x) + t_d \tag{2}$$

where

$$t_T(100) = T \tag{3}$$

We introduce the transformation linking the data covered by equations (1) and (2). Thus

$$t_{T0}(x) = F \cdot t_{W0}(x) \tag{4}$$

where

$$F = t_{T0}(100)/t_{W0}(100) \tag{5}$$

and

$$t_{T0}(100) = T - t_d \tag{6}$$

As previously indicated, to two decimal places, $t_W(100) = 10.16$ s, $t_{W0}(100) = 9.98$ s and $t_d = 0.18$ s. Running speeds are scaled according to the relationship:

$$v_{T0}(x) = v_{W0}(x)/F \tag{7}$$

The main studies were based on 100 m performances with $T = 10$ s. However, the effects on relay performance of slower runners, with $T = 10.1$ s, 10.2 s and 10.3 s, were also investigated. The variations of $t_{T0}(x)$ and $v_{T0}(x)$ with x were calculated at 1 m intervals along the track.

It is normal practice at the handover of the baton for the incoming athlete to present the baton by reaching forwards and for the outgoing runner to reach backwards to receive the baton. Consequently, at the exchange, the incoming and outgoing runners are normally a certain distance apart, known as the 'free distance'. For convenience, we shall define the free distances at the end of legs 1, 2 and 3 by x_{f1}, x_{f2} and x_{f3}, respectively, and the total free distance $(= x_{f1} + x_{f2} + x_{f3})$ by x_{fT}. The exploitation of free distance at the baton exchanges is important in two respects. First, it leads to a direct improvement in the running time of the team. Secondly, it ensures that the incoming and outgoing runners do not run into each other or otherwise impede each other's movements.

Once the performance of the four individual athletes is known, the performance of the relay team can be calculated, provided information is available on the following: (a) the starting positions of the athletes on each leg, (b) the positions at which baton exchanges occur, (c) the free distance between the incoming and outgoing athletes at each of the baton exchanges and (d) the running order of the athletes.

Track geometry

The geometry of the IAAF 400 m standard track, with eight lanes, is specified by the IAAF (1999) in its *Track and Field Facilities Manual*. Across the eight lanes, the positions of the exchange zones for the 4×100 m relay are staggered (Table 1). The stagger is the distance of the scratch line defining the start of each leg from the beginning or end of the associated bend. The two bends in each lane have the same radius of curvature and this dimension is defined for a theoretical line of running along which it is assumed an athlete moves.

Track curvature effects

To determine the effects of track curvature on running performance, we followed the theoretical work of Greene (1985), who used Newton's laws to investigate the relationship between the components of the average ground-contact force when a runner moves along a curved path. The theory of Greene (1985) was derived assuming a constant top speed of the sprinter. However, this restriction is not essential and the relationship between running speeds on straight and curved tracks derived by Greene (1985) is equally applicable to accelerated or decelerated running. This is because the relationship depends on the transverse forces acting on the runner, which are fully determined by the instantaneous speed along the track and the radius of curvature of the running path. Here, the results of

Table 1. Geometry of the IAAF 400 m standard track

Lane	Bend radius (m)	Stagger (m)			
		Leg 1	Leg 2	Leg 3	Leg 4
1	36.80	0	−15.61	0	−15.61
2	37.92	7.04	−12.09	3.52	−15.61
3	39.14	14.70	−8.26	7.35	−15.61
4	40.36	22.37	−4.43	11.19	−15.61
5	41.58	30.03	−0.59	15.02	−15.61
6	42.80	37.70	3.24	18.85	−15.61
7	44.02	45.37	7.07	22.68	−15.61
8	45.24	53.03	10.91	26.52	−15.61

Note: The bend radius and stagger at the start of each leg in the 4×100 m relay are given. The stagger is the distance of the scratch line defining the start of the leg relative to the beginning or end of the associated bend.

Greene's analysis are applied to the range of instantaneous speeds that occur in sprinting.

Consider a sprinter, whose speed running on a straight track would be v_0, but who is in fact running on a curved path, with radius of curvature R, such that his running speed is reduced to v. Following Greene (1985), we define the non-dimensional variables

$$w = (v/v_0)^2 \qquad (8)$$

and

$$r = Rg/v_0^2 \qquad (9)$$

where g is the gravitational acceleration. Greene's theory shows that w and r are related by the equation

$$w = \left[\frac{r^2}{2} + \sqrt{\frac{r^4}{4} + \frac{r^6}{27}} \right]^{1/3} + \left[\frac{r^2}{2} - \sqrt{\frac{r^4}{4} + \frac{r^6}{27}} \right]^{1/3} \qquad (10)$$

From equation (8):

$$v = v_0\sqrt{w} \qquad (11)$$

For elite male runners, the smallest values for r encountered on the IAAF track geometry are about 3, so that the effect of track curvature on running speed is typically 5% or less. Greene (1985) did not undertake any tests on the synthetic surfaces typical of modern athletic stadia; however, for values of r of 3 and higher, he showed that there was good agreement between theory and experimental results for running on flat concrete surfaces.

Equations (8) to (11) were used to derive the running profile around the bend, yielding the running speed at 1 m intervals. In other words, the relationship between v and x round the bend, suitably adjusted for curvature

effects, is known. The relationship between t and x was obtained by numerical integration of the expression

$$t(x) = \int_0^x dx/v \qquad (12)$$

Equation (12) has a singularity at $t = 0$, $v = 0$. As v tends to zero, curvature has a vanishingly small effect on the relationships between t and x and v; therefore, to avoid problems with the singularity, the straight-line data were used for the first interval of the integration process.

Results

The analysis is organized as follows. First, we consider a running profile representative of the performance of an elite male sprinter. Next, we set out a nominal schedule for the 4×100 m relay, giving the progress of the baton along the track and the distance covered by each of the four runners during their individual legs of the race, assuming all four athletes have the same nominal performance over 100 m. We then consider the time course of the baton and the speed of the runners at each baton exchange. These early calculations are intended to reveal some initial insights into relay running, but do not take account of the effect of track curvature on running speed. Subsequent calculations incorporate this factor and show it to have an important bearing on running time. The effects on relay time of altering (1) the positions from which runners start their legs and (2) the position of the baton exchange are then analysed. Another factor considered is the free distance between the athletes at the baton exchanges. For the representative running profile considered, we try to determine the optimum positions from which the runners start each leg and at which the baton exchanges occur. Finally, we try to determine the consequences for overall running time in a 4×100 m relay of including athletes with different running performances over 100 m.

A running profile representative of the performance of an elite male sprinter is given in columns 2, 3 and 4 of Table 2. This running profile was derived using the mathematical model of Ward-Smith and Radford (2000a) and, in effect, is representative of the overall average performance of the 100 m finalists at the 1987 World Championships adjusted to zero wind conditions. In column 2 a zero reaction time is assumed, while in column 3 a reaction time of 0.18 s at the initiation of sprinting has been included. The figure of 0.18 s was the average reaction time of the finalists in the 100 m event at the 1987 World Championships (Moravec *et al.*, 1988) and is used throughout this study. The calculations, which are based on the biophysical parameters proposed by Ward-Smith and Radford (2000a) and assume sprinting with maximum effort throughout, have been extended beyond 100 m up to 130 m, the

Table 2. Representative running performance of an elite male sprinter

Distance (m)	Based on Ward-Smith and Radford			Adjusted performance		
	Time (s)	Time (s)	Speed (m·s^{-1})	Time (s)	Time (s)	Speed (m·s^{-1})
0	0	0.18	0	0	0.18	0
10	1.75	1.93	8.70	1.72	1.90	8.85
20	2.80	2.98	10.11	2.76	2.94	10.28
30	3.76	3.94	10.79	3.70	3.88	10.97
40	4.67	4.85	11.16	4.59	4.77	11.35
50	5.55	5.73	11.35	5.46	5.64	11.53
60	6.43	6.61	11.41	6.33	6.51	11.60
70	7.31	7.49	11.39	7.19	7.37	11.57
80	8.19	8.37	11.30	8.06	8.24	11.49
90	9.08	9.26	11.17	8.93	9.11	11.35
100	9.98	10.16	11.01	9.82	10.00	11.19
110	10.90	11.08	10.83	10.72	10.90	11.00
120	11.83	12.01	10.63	11.64	11.82	10.81
130	12.78	12.96	10.43	12.57	12.75	10.60

Note: The figures in columns 2–7 have been rounded to two decimal places. Columns 2 and 5 assume zero reaction time. Columns 3 and 6 take reaction time = 0.18 s. Columns 2, 3 and 4 are based on, and consistent with, Ward-Smith and Radford (2000a,b) for zero wind velocity.

Table 3. Progress of the runners and baton along the track during the four legs of a reference 4 × 100 m relay

Leg	Runner movement (m)			Baton movement (m)		
	Start	Distance run	Finish	Start	Increment	Finish
1	0	100	100	0	100	100
2	80	120	200	100	100	200
3	180	120	300	200	100	300
4	280	120	400	300	100	400

Note: Distances are measured from the start of the race.

longest distance an athlete can cover in a 4 × 100 m relay. In columns 5, 6 and 7, a separate set of performance data is given. The data in column 5 were derived from column 2 by multiplying all terms in column 2 by a common factor F (see equations 4 and 5). That same quantity was used to divide the numbers in column 4 to derive column 7. The factor F was chosen such that, for zero reaction time, in column 5 the time to 100 m is 9.82 s. This yields a running performance in column 6, which, when the reaction time of 0.18 s is added on, gives a 100 m performance of 10.00 s.

A nominal schedule for the 4 × 100 m relay is set out in Table 3, giving the progress of the baton along the track and the distance covered by each of the four runners during their individual legs of the race. For the purpose of these calculations, apart from the first leg, each member of the team was assumed to start running from a point 10 m before the takeover zone, the maximum distance outside the takeover zone permitted by the regulations. We assumed that, at the end of each leg, the handover of the baton takes place on the scratch line, so that all four athletes carry the baton 100 m. For simplicity, in the initial calculations the free distance is taken as zero.

The time course of the baton and the speed of the runners at each baton exchange are given for a reference relay in Table 4. All four runners were assumed (a) to have the running profile defined in columns 5–7 of

Table 4. Progress of the runners and baton along the track during the four legs of a reference 4×100 m relay

Leg	Baton movement (s)			Speed of runner ($\text{m} \cdot \text{s}^{-1}$)	
	Start	Increment	Finish	Start	End
0*	0	0.18	0.18	0	0
1	0.18	9.82	10.00	0	11.19
2	10.00	8.88	18.88	10.28	10.81
3	18.88	8.88	27.76	10.28	10.81
4	27.76	8.88	36.64	10.28	10.81

Note: All the athletes are assumed to have the nominal performance specified in columns 5–7 of Table 2 and to follow the schedule defined in Table 3. Times are measured from the start of the race. Also shown are the running speeds of the athletes at the start and finish of the race and at the baton exchange at the beginning and end of each leg.
* A reaction time delay, taken as 0.18 s, affects the first runner but not the others.

Table 5. Geometry of race track and distances run round the bend (B) and along the straight (S), for the eight lanes and four legs of each race, assuming the running schedule set out in Table 3

Lane	Radius of curvature	Distances run (m)									
		Leg 1		Leg 2			Leg 3			Leg 4	
		B	S	B	S	B	S	B	B	S
1	36.80	100	0	35.61	84.39	0	20	100	35.61	84.39
2	37.92	100	0	32.09	84.39	3.52	16.48	103.52	35.61	84.39
3	39.14	100	0	28.26	84.39	7.35	12.65	107.35	35.61	84.39
4	40.36	100	0	24.43	84.39	11.19	8.82	111.18	35.61	84.39
5	41.58	100	0	20.59	84.39	15.02	4.98	115.02	35.61	84.39
6	42.80	96.76	3.24	16.76	84.39	18.85	1.15	118.85	35.61	84.39
7	44.02	92.93	7.07	12.93	84.39	22.68	0	120	35.61	84.39
8	45.24	89.09	10.91	9.09	84.39	26.52	0	120	35.61	84.39
Total distance		100		120			120		120	

Table 2 and (b) to follow the schedule set out in Table 3. In these calculations, only the athlete running the first leg was assumed to suffer a reaction time delay, taken as 0.18 s.

Table 5 provides, for the running schedule set out in Table 3, the distances run round the bend and along the straight for the eight lanes and four legs of each race. Segments run round the bend are indicated by B and along the straight by S. Table 5 identifies an important advantage conferred on the teams drawn on the outside lanes; the radius of curvature of the bends is larger for these lanes. Table 5 reveals a second effect. The total distance covered by the athletes in each team exceeds 400 m. The proportion of that total distance run along straight sections of the track varies from lane to lane, the figure depending on the locations of the baton exchanges and the position from which the athletes on legs 2, 3 and 4 start running.

The effect of bend curvature on running time for the 4×100 m relay is shown in Table 6. The results were obtained using equations (8) to (12) and take account of the continuously changing speed of the runners through the bend. We assumed that the athletes use the schedule set out in Table 3 and the calculations were based on four sprinters each capable of running 100 m in 10 s. The free distance at the baton exchanges was taken as zero. The results are given for all eight lanes and may be compared with a relay time uncorrected for bend curvature effects of 36.64 s.

Until now, all of the relay calculations have been based on the schedules set out in Table 3, which gives the positions from which the runners start each leg and the positions of baton exchanges. The schedule set out in Table 3 is non-optimal; lower overall relay times are possible with other schedules for teams composed of four sprinters capable of running 100 m in 10 s. The

effects of altering the position of the baton exchanges and the starting positions of the runners in legs 2, 3 and 4 are considered in Table 7. Bend curvature effects were taken into account and the free distance at the baton exchanges was taken as zero. The starting conditions for the first leg are evidently fixed. Several new schedules are included in Table 7 and the calculations are applied

Table 6. Effect of bend curvature on running time for the 4 × 100 m relay

Lane	Bend radius (m)	Running time (s)
1	36.80	37.60
2	37.92	37.56
3	39.14	37.51
4	40.36	37.47
5	41.58	37.43
6	42.80	37.39
7	44.02	37.36
8	45.24	37.33

Note: Calculations are based on four sprinters each capable of running 100 m in 10 s, running according to the schedule in Table 3. The free distance is taken as zero. Relay time uncorrected for bend curvature effects = 36.64 s.

to lanes 1 and 8. So that the effects of altering the parameters are clear, cases 1 and 10 – for lanes 1 and 8 respectively – are based on the schedule set out in Table 3. Cases 2–4 and 11–13 show how the starting position of the outgoing runners affects running time, while the positions of the baton exchanges remain unchanged. Cases 5–7 and 14–16 show the effect on running time of altering the positions of the baton exchanges, while the starting positions of the outgoing runners remain unaltered. Cases 8 and 9, and 17 and 18, give the results for other combinations of positions of the variables. In particular, case 9 represents an optimum schedule for lane 1 and case 18 provides the corresponding results for lane 8.

The free distance between runners at the baton exchanges has so far been taken as zero. At this stage, we take account of this factor and include a finite distance between the runners at the baton exchange. The results, for the running schedule given in Table 3, are given in Table 8, where results for lanes 1–8 are included. For each group of calculations, we assumed that the same free distance occurs at all three baton exchanges during any relay. The results in Table 8 can be compared with those of Table 6, which are for zero free distance, and

Table 7. The influence of the runner's starting position in legs 2, 3 and 4 and the position of the baton exchange on overall running time

		Starting position (m)						
		Runner			Baton			
Case	Lane	Leg 2	Leg 3	Leg 4	Leg 2	Leg 3	Leg 4	Time (s)
1	1	80	180	280	100	200	300	37.60
2	1	84	184	284	100	200	300	37.68
3	1	88	188	288	100	200	300	37.82
4	1	90	190	290	100	200	300	37.93
5	1	80	180	280	102	202	302	37.58
6	1	80	180	280	106	206	306	37.55
7	1	80	180	280	110	210	310	37.56
8	1	84	184	284	106	206	306	37.57
9*	1	84	182	280	110	208	306	37.54
10	8	80	180	280	100	200	300	37.33
11	8	84	184	284	100	200	300	37.41
12	8	88	188	288	100	200	300	37.56
13	8	90	190	290	100	200	300	37.67
14	8	80	180	280	102	202	302	37.30
15	8	80	180	280	106	206	306	37.27
16	8	80	180	280	110	210	310	37.28
17	8	84	184	284	106	206	306	37.29
18*	8	84	182	280	110	208	306	37.26

Note: The starting positions of the runner and baton on leg 1 are at $x = 0$. All starting positions on legs 2, 3 and 4 are measured relative to the starting line, $x = 0$. The effect of bend curvature on running speed is taken into account. The free distance at baton exchanges is taken as zero.
* The best time for the lane.

Table 8. The effect of free distance on running time at each of the baton exchanges

Case	Lane	Free distance (m)	Time (s)
1	1	1	37.31
2	2	1	37.26
3	3	1	37.21
4	4	1	37.17
5	5	1	37.14
6	6	1	37.10
7	7	1	37.06
8	8	1	37.03
9	1	2	37.00
10	2	2	36.96
11	3	2	36.91
12	4	2	36.87
13	5	2	36.84
14	6	2	36.80
15	7	2	36.77
16	8	2	36.74

Note: The running schedule of Table 3 is used. The effect of bend curvature on running speed is taken into account. Results for zero gap are contained in Table 6.

Table 9. The effect of different free distances at each of the baton exchanges

Lane	Free distance (m)			Time (s)
	Leg 1	Leg 2	Leg 3	
1	1	1	1	37.31
1	0	1	2	37.31
1	0	2	1	37.31
1	1	0	2	37.31
1	1	2	0	37.31
1	2	0	1	37.31
1	2	1	0	37.31
8	1	1	1	37.03
8	0	1	2	37.03
8	0	2	1	37.03
8	1	0	2	37.03
8	1	2	0	37.03
8	2	0	1	37.03
8	2	1	0	37.03

Note: Total free distance = 3 m. Calculations based on four sprinters each capable of running 100 m in 10 s, running according to the schedule in Table 3. The effect of bend curvature on running speed is taken into account.

demonstrate that any free distance acquired at the baton exchanges has a significant effect, reducing the overall time taken by a team to complete a 4×100 m relay. If a free distance of 1 m occurs at all three baton exchanges (i.e $x_{f1} = x_{f2} = x_{f3} = 1$ m), then the total free distance x_{fT} is 3 m. Table 9 shows the results of the effect on overall running time of different free distances x_{f1}, x_{f2} and x_{f3} at the baton exchanges, for a given total free distance x_{fT}. Results are given in Table 9 for lanes 1 and 8, with an overall free distance x_{fT} of 3 m. Tables 8 and 9 show that the total free distance has an important influence on the overall time to complete the relay but, for a given total free distance, the distribution of the individual free distances at the three baton exchanges has no effect.

All of the preceding calculations of relay completion times were carried out for specified running schedules. The starting and end positions of the athletes in all four legs were specified. In the final set of calculations, that restriction was relaxed. In Table 10, the optimum running schedules, corresponding to the lowest running time, are set out for lanes 1–8, for three different free distances (identified as 'gap' in the table) at each of the baton exchanges. In all of the cases considered, $x_{f1} = x_{f2} = x_{f3}$. The runner's starting position and end position, at the baton exchange, on legs 1, 2, 3 and 4 are shown. All distances on legs 2, 3 and 4 were measured relative to the starting line, $x = 0$. The effect of bend curvature on running speed was taken into account.

Until now, the calculations have been based on four athletes with identical performances over 100 m. Tables

11 and 12 show the consequences for overall running time in a 4×100 m relay of including athletes with different running performances over 100 m. The calculations are based on the running schedule set out in Table 10 (cases 9–13) and involve athletes with nominal 100 m performances of 10 s, 10.1 s, 10.2 s and 10.3 s respectively, including a reaction time of 0.18 s in all cases. These 100 m performances were calculated using equations (1) to (5). Table 11 sets out 28 different cases; the running times for each case are given for lanes 1, 3, 5 and 8 in Table 12. The first four rows of Tables 11 and 12 show the overall running time for four athletes with identical performances. Rows 5–28 show the results of assigning athletes with different performances over 100 m to each of the four legs. In calculating the overall relay time, only the athlete running the first leg was assumed to suffer a delay (taken as 0.18 s) due to reaction effects. Rows 1–4 show clearly the benefits of having fast runners in the relay. Not surprisingly, a relay team composed of four athletes capable of completing 100 m in 10 s (9.82 s plus 0.18 s reaction time) can complete the 4×100 m relay over 1 s faster than a team of four 10.3 s (10.12 s plus 0.18 s) runners.

Discussion

In interpreting the following discussion, it is helpful at the outset to set out an approximate relationship

Table 10. The optimum running schedules for lanes 1–8, showing the effect of free distance (identified as 'Gap') at each of the baton exchanges

| | | | Start and end position of runner on legs 1, 2, 3 and 4 (m) | | | | | | | | |
| | | | Leg 1 | | Leg 2 | | Leg 3 | | Leg 4 | | |
Case	Lane	Gap (m)	S	E	S	E	S	E	S	E	Time
1	1	0	0	110	84	208	182	306	280	400	37.54
2	2	0	0	110	84	208	182	306	280	400	37.50
3	3	0	0	110	84	208	182	306	280	400	37.46
4	4	0	0	110	84	208	182	306	280	400	37.41
5	5	0	0	110	84	208	182	306	280	400	37.37
6	6	0	0	110	84	208	182	306	280	400	37.33
7	7	0	0	110	84	208	182	306	280	400	37.29
8	8	0	0	110	84	208	182	306	280	400	37.26
9	1	1	0	109	84	207	182	305	280	400	37.25
10	2	1	0	109	84	207	182	305	280	400	37.21
11	3	1	0	109	84	207	182	305	280	400	37.17
12	4	1	0	109	84	207	182	305	280	400	37.13
13	5	1	0	109	84	207	182	305	280	400	37.09
14	6	1	0	109	83	206	181	305	280	400	37.05
15	7	1	0	109	83	206	181	305	280	400	37.01
16	8	1	0	109	83	206	181	305	280	400	36.98
17	1	2	0	109	85	207	182	304	280	400	36.96
18	2	2	0	109	84	206	182	304	280	400	36.92
19	3	2	0	109	84	206	182	304	280	400	36.88
20	4	2	0	109	84	206	182	304	280	400	36.84
21	5	2	0	109	84	206	182	304	280	400	36.80
22	6	2	0	109	84	206	182	304	280	400	36.76
23	7	2	0	109	84	206	182	304	280	400	36.73
24	8	2	0	109	84	206	182	304	280	400	36.70

Note: The runner's starting (S) position and end (E) position, at the baton exchange, on legs 1, 2, 3 and 4 are shown. All distances on legs 2, 3 and 4 are measured relative to the starting line, $x = 0$. The effect of bend curvature on running speed is taken into account.

between time gained or lost and the corresponding margin at the finishing line. The average speed at which an elite team of male athletes completes the 4×100 m relay is just over 10 m·s^{-1}. Hence an improvement of running time of 0.1 s equates to just under 1 m at the finishing line, 0.01 s equates to just under 0.1 m, and so on.

As a context for the discussion of the computed results, it is appropriate to document relevant world records on the track, current at the time of writing. The men's 100 m world record is 9.79 s, set by Maurice Greene (USA) on 16 June 1999 in Athens. There was an assisting wind of 0.1 m·s^{-1}. Adjusting for wind effects (Linthorne, 1994; Ward-Smith, 1999), the basic 9.79 s is increased by 0.005 s, which to two decimal places leaves the original figure unchanged. The men's 4×100 m world record is 37.40 s, set on 8 August 1992 by the US team at the 1992 Olympics in Barcelona. It is

interesting to note that the relay time of 37.40 s is faster than four times the 100 m record ($4 \times 9.79 = 39.16$) by 1.76 s, a margin of some 4.7%. The present calculations show that many factors affect the relay time, including the 100 m times of the individual runners, the legs to which the four athletes are assigned, the lane draw, the positions from which each of the four athletes start running, the position of the baton exchange and the total free distance accumulated at the baton exchanges. Most of the calculations reported here are based on a relay team composed of four elite male athletes, each capable of running the 100 m in 10 s. Table 10 shows that, for a typical total free distance of 3 m and with the team running in one of the centre lanes, the fastest time in which these four athletes could complete the relay is between 37.05 s and 37.13 s, depending on lane draw. These figures, which account for bend curvature effects, compare very well with the world record of 37.40 s.

Table 11. The influence of performance over 100 m on the overall running time (s) for the 4 × 100 m relay

| | Running time (s) | | | |
Case	Leg 1	Leg 2	Leg 3	Leg 4
1	10.0	10.0	10.0	10.0
2	10.1	10.1	10.1	10.1
3	10.2	10.2	10.2	10.2
4	10.3	10.3	10.3	10.3
5	10.0	10.1	10.2	10.3
6	10.0	10.2	10.1	10.3
7	10.0	10.1	10.3	10.2
8	10.0	10.3	10.1	10.2
9	10.0	10.3	10.2	10.1
10	10.0	10.2	10.3	10.1
11	10.1	10.0	10.2	10.3
12	10.1	10.2	10.0	10.3
13	10.1	10.0	10.3	10.2
14	10.1	10.3	10.0	10.2
15	10.1	10.2	10.3	10.0
16	10.1	10.3	10.2	10.0
17	10.2	10.0	10.1	10.3
18	10.2	10.1	10.0	10.3
19	10.2	10.0	10.3	10.1
20	10.2	10.3	10.0	10.1
21	10.2	10.1	10.3	10.0
22	10.2	10.3	10.1	10.0
23	10.3	10.0	10.1	10.2
24	10.3	10.1	10.0	10.2
25	10.3	10.0	10.2	10.1
26	10.3	10.2	10.0	10.1
27	10.3	10.1	10.2	10.0
28	10.3	10.2	10.1	10.0

Note: This table outlines the cases for which detailed data are given in Table 12. The first four rows are cases for four athletes with identical performances. For cases 5–28, athletes with different performances over 100 m are assigned to each of the four legs. The four athletes have nominal 100 m performances of 10.0 s, 10.1 s, 10.2 s and 10.3 s, respectively, including a reaction time of 0.18 s in all cases. Only the athlete running the first leg is assumed to suffer a delay (taken as 0.18 s) due to reaction effects. Allowance is made for track curvature effects.

Table 12. The influence of performance over 100 m on the overall running time for the 4 × 100 m relay

| | Running time (s) | | | |
Case	Lane 1	Lane 3	Lane 5	Lane 8
1	37.25	37.17	37.09	36.98
2	37.60	37.50	37.42	37.31
3	37.94	37.85	37.76	37.66
4	38.28	38.19	38.11	38.01
5	37.75	37.66	37.57	37.47
6	37.76	37.66	37.58	37.47
7	37.74	37.65	37.56	37.46
8	37.76	37.67	37.58	37.47
9	37.76	37.66	37.57	37.47
10	37.75	37.65	37.57	37.46
11	37.76	37.66	37.58	37.48
12	37.77	37.68	37.59	37.49
13	37.75	37.66	37.57	37.47
14	37.78	37.68	37.59	37.49
15	37.76	37.66	37.58	37.48
16	37.77	37.67	37.58	37.48
17	37.77	37.68	37.60	37.50
18	37.78	37.69	37.60	37.50
19	37.76	37.67	37.59	37.49
20	37.79	37.69	37.61	37.50
21	37.76	37.67	37.59	37.49
22	37.78	37.69	37.60	37.50
23	37.78	37.69	37.61	37.51
24	37.79	37.70	37.61	37.51
25	37.78	37.69	37.60	37.51
26	37.79	37.70	37.62	37.52
27	37.78	37.69	37.60	37.51
28	37.79	37.70	37.61	37.51

Note: Table 11 shows the time over 100 m of the athletes assigned to each leg. It is assumed that the start and end positions of the athletes is as for the schedule set out in Table 10, cases 9–13. A free distance of 1 m is assumed at each baton exchange. The first four rows show the overall running time for four athletes with identical performances. Rows 5–28 give the result of assigning athletes with different performances over 100 m to each of the four legs. Times for lanes 1, 3, 5 and 8, corrected for bend curvature, are shown.

Again they are significantly faster than four times the 100 m times (4 × 10 s = 40 s). The difference between the predicted times of 37.05 s and 37.13 s and the world record of 37.40 s is less than 1%, and it is not surprising to find that the predicted times are slightly less than the figure from competition. In the calculations, the starting positions of the runners on the final three legs were optimized and baton exchanges were assumed to take place at or close to the extreme limits permitted by the rules. On the track, while the athletes can afford to run flat out over most of each leg, such a completely uninhibited approach is inappropriate at the baton exchanges because the successful completion of the transfer process could be put at risk. An increase in running time as small as (37.4 − 37.1)/3 = 0.1 s at each exchange is sufficient to bring the two sets of figures into line, and can be explained as an inevitable consequence of the care exercised by both the incoming and outgoing athletes to secure the successful transfer of the baton and to avoid disqualification by overrunning the exchange zone. In summary, the good agreement

between the competition and predicted data allows us to draw conclusions from the detailed predictions of this study with considerable confidence.

There are two principal reasons for the relay time being lower than the sum of the 100 m times of the four athletes. The data in Tables 2–4 provide an explanation for one of the factors. Table 2 shows that an athlete requires at least 20 m to reach a speed above $10 \text{ m} \cdot \text{s}^{-1}$. This acceleration phase causes the average speed over 100 m to be well below the instantaneous speeds attained over the remaining cruise phase of the race. In the relay, the first leg contains an acceleration phase similar to that of a 100 m race but, in the subsequent three legs, the baton is carried by athletes who have already attained full running speed. In these final three legs, the average speed at which the baton is carried is much higher than the average speed of a 100 m race. For the running schedule set out in Table 3, each athlete carries the baton for a distance of 100 m. Although the athlete running the first leg takes 9.82 s to carry the baton forward, the final three athletes only need 8.88 s, thereby ensuring that, overall, the average speed in the 4×100 m relay is greater than that of the 100 m race. A second factor further reduces the relay time. The opportunity to create a free distance between the incoming and outgoing athletes at the baton exchanges saves running time in a way that is not available during the 100 m sprint.

The calculations show that bend curvature effects have an important role in determining the overall relay time. For a relay team composed of four runners capable of completing 100 m in 10 s running according to the schedule set out in Table 3 and with no free distance at the baton exchanges, the computed time, unadjusted for the effects of bend curvature, is 36.64 s (Table 4). When bend curvature effects are taken into account, running times increase. But Table 6 clearly shows that the effect is not uniform across all eight lanes. The effect is smallest for lane 8 and increases progressively as the lane number reduces. For lane 8, the computed running time is 37.33 s; for lane 1, it is 37.60 s. Thus bend curvature effects add a minimum of 0.69 s for lane 8, rising to 0.96 s for lane 1. The difference across the eight lanes is 0.27 s, which is equivalent to some 3 m difference at the finishing line.

Table 7 contains several results showing the effects of altering the positions of the baton exchanges and the starting positions of the runners in legs 2, 3 and 4, evaluated for lanes 1 and 8. All of the athletes are assumed to have the running profile shown in columns 5–7 of Table 2, suitably modified for bend curvature effects. In particular, case 9 represents an optimum schedule for lane 1 and case 18 provides the corresponding results for lane 8. Table 7 also shows that, if the wrong choices are made for the positions of the baton exchanges and the starting positions of the runners, then overall running time is increased.

The effect of free distance at the baton exchanges is shown in Table 8. The calculations assume the same free distance between the incoming and outgoing athlete at each of the three baton exchanges. Free distances of 0, 1 and 2 m, corresponding to total free distances of 0, 3 and 6 m, are considered. The results show that exploiting the opportunities to gain free distance is an important factor in determining the time in which a team completes a 4×100 m relay. Further calculations, set out in Table 9 and based on a total free distance of 3 m, show the effect of having different free distances at each of the baton exchanges. These calculations show that the time to complete the relay is determined by the total free distance and is unaffected by how the individual components aggregate to give the total figure. We recognize that free distances of 2 m at the baton exchanges are probably not achievable while running at top speed. The relevance of these calculations is that they allow interpolation for intermediate values of total free distance. From Tables 6 and 8, a total free distance of 3 m is worth approximately 0.3 s across lanes 1–8, while 6 m is worth roughly twice that amount. The time saved is essentially proportional to the total free distance. As a useful rule of thumb, each 1 m of total free distance reduces the time for the team to complete the relay by just under 0.1 s.

Most of the calculations reported here have been for specified running schedules, such as those set out in Tables 3 and 7. In specifying both the representative running performance and the running schedule, the speed of the incoming and outgoing runners at the baton exchange is predetermined. This generally means that the speeds of the two runners are similar but unequal, as shown in Table 4 for example. The outgoing runner shows an increasing speed profile at the baton exchange; the incoming runner, having covered more than 100 m, shows a slowly decreasing speed profile. The calculations in Table 10 determine the optimum runners' starting positions and positions of the baton exchanges, calculated across the eight lanes and for a range of free distances at the baton exchanges and correspond to the lowest times for the completion of the relay. For a typical total free distance of 3 m, the optimum schedules involve the runner of leg 2 starting at $x = 84$ m or $x = 83$ m and the runner of leg 3 starting at $x = 182$ m or $x = 181$ m. The start of the fourth leg at $x = 280$ m is the same for both the optimum and reference schedules. For the optimum schedules, the athletes in legs 1, 2, 3 and 4 run distances of approximately 109, 123, 124 and 120 m respectively, carrying the baton 110, 98, 98 and 94 m respectively. Detailed calculations (not included here) show that, for the optimum schedules, the running speeds of the incoming and

Table 13. Approximate indication of the relative importance of factors affecting relay time

Factor	Time increment (s)	Comment
Lane draw Reduction in running time for lane 8 compared with lane 1	0.27–0.28	This factor has an important bearing on the outcome of a relay race, but is outside the control of the team coach
Running order The effect on running time of changing the order in which four athletes capable of completing 100 m in 10.0, 10.1, 10.2 and 10.3 s respectively are assigned to the four legs	0–0.06	This factor is within the control of the team coach
Starting position of runners on legs 2, 3 and 4 The effect of making minor adjustments to the starting positions, assuming the athletes start running at least 6 m ahead of the exchange zone	0–0.08	This factor is within the control of the individual athlete and team coach
Position of baton exchange The effect of minor adjustments, assuming the baton is exchanged near the exit of the exchange zone	0–0.06	This factor is within the control of the athletes and team coach
Free distance Total free distance taken as 3 m	0–0.30	This factor is within the control of the athletes and the team coach
Bend curvature effects Increase in running time on a curved track compared with running the relay on a straight track	0.69–0.96	Of theoretical interest only

outgoing runners are equal at the baton exchanges at the end of the second and third legs. At the first exchange, the speed of the incoming athlete always exceeds that of the outgoing runner.

Rows 4–28 of Tables 11 and 12 show calculations for a relay team composed of four individuals with different performances over 100 m. The team is assumed to be made up of one member from each of the groups capable of completing 100 m in 10 s, 10.1 s, 10.2 s and 10.3 s. We assumed a free distance of 1 m at each baton exchange, together with the running schedule shown in Table 10, cases 9–13. The overall running time of the team was shown to depend to some extent on the leg to which each member was assigned. The differences in overall running time between one arrangement and another were shown to be small but significant, differing by 0.06 s between the fastest and the slowest. Across all four lanes, case 7 yielded the fastest relay time, while case 26 yielded the slowest. These results show that the best times are achieved by allocating the fastest athlete to the first leg and using the slower athletes on legs 3 and 4. As a simple rule of thumb, an arrangement close to the optimal has the fastest runner in leg 1, followed by the next fastest runner in leg 2, the next fastest runner in leg 3 and the slowest runner assigned to the final leg.

In Table 13, we try to summarize the effects of the various factors that affect overall relay time. Factors over which the individual athletes, and the team coach, can exercise some control are: the free distances at the baton exchanges, the running order of the athletes, the starting positions of the runners on legs 2, 3 and 4, and the positions at which baton exchanges occur. The lane draw has an important influence on relay time, but is outside the control of the team coach. It is of theoretical interest to note that, if the 4×100 m relay could be run on a straight track, relay times could be improved by between 0.69 s and 0.96 s.

We have shown that, for teams composed of elite male athletes, bend curvature effects can confer an advantage of some 0.27 s on the team drawn in lane 8 compared to that drawn in lane 1. This figure can be put in perspective by noting that, in a 100 m race, an assisting wind of 2 $m \cdot s^{-1}$ reduces the running time of a 10 $m \cdot s^{-1}$ sprinter by 0.1 s (Linthorne, 1994; Ward-Smith, 1999), and that efforts are made to measure wind speed so that records can be disallowed if the wind speed exceeds 2 $m \cdot s^{-1}$. For championships involving elite athletes,

such as the World Championships and Olympic Games, a serious case could be made to compensate properly for the variation of bend curvature effects across the eight lanes. However, at the same time, it must be recognized that the differential across the eight lanes is a function of running speed, with the consequence that non-elite athletes are not so severely disadvantaged by this effect.

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