Contributions of Angular Momentum and Catting to the Twist Rotation in High Jumping

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This project sought to break down high jump twist rotation into portions contributed by angular momentum and those contributed by rotational action and reaction ("catting"). Five male and 5 female high jumpers were studied with three-dimensional film/video analysis procedures. The hip twist angle at the peak was broken down into an initial twist angle at takeoff and the subsequent twist rotation accumulated between takeoff and the peak. The latter was in turn broken down into rotations contributed by the twisting component of angular momentum and rotations contributed by catting. It was found that the contribution of catting to the twist rotation was at least as large as that of the angular momentum. The important contribution of catting to the twist rotation introduces the possibility that defects in its execution might play a role in the problems that some high jumpers have with twist rotation.

During the takeoff of a high jump, the athlete exerts forces on the ground which determine the maximum height that the center of mass (CM) will reach and the angular momentum of the body (Dapena, 1980a, 1980b; Dyson, 1970). The jumper makes a twisting somersault rotation in the air that leads to a supine layout position at the peak of the jump (Figure 1a), but problems in the twist rotation can produce a tilted position, with one hip lower than the other (Figure 1b). The low hip limits the result of the jump. To correct problems in the twist rotation of high jumpers, we need to improve our understanding of the mechanisms that produce twist rotation.

To some extent, a high jumper’s rotations are similar to those of a rigid circular cylinder. If the cylinder’s longitudinal axis is oblique to the angular momentum vector $\textbf{H}$, the latter can be separated into two components: $\textbf{H}_s$, normal to the longitudinal axis, and $\textbf{H}_t$, aligned with the longitudinal axis (Figure 2a). The longitudinal axis will precess, tracing out a conical surface in the air, while the cylinder spins about the longitudinal axis (Figure 2b). This motion can be described as a twisting somersault. $\textbf{H}_s$ produces the precession or somersault rotation and $\textbf{H}_t$, the spin or twist rotation (Frohlich, 1980; Hopper, 1973). Figure 2c shows a typical high jumper at the end of the takeoff and the constant angular momentum vector that the athlete will have in the air (Dapena, 1980b). The longitudinal principal axis is oblique to the angular momentum vector, which will tend to make the athlete somersault and twist in the air like the cylinder. However, the human body is not rigid, which introduces differences with respect to the cylinder.

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Glossary

CM  center of mass
H  angular momentum vector
\( H_s \)  somersaulting component of angular momentum; the projection of \( H \) on the plane normal to the longitudinal axis of the body
\( H_t \)  twisting component of angular momentum; the projection of \( H \) on the longitudinal axis of the body
\( H_{t,avg} \)  average twisting angular momentum in the interval between two frames
\( H_{t,avg} \)  average twisting angular momentum between takeoff and the peak
\( H_{t,init} \)  twisting angular momentum at takeoff
\( \Delta H_t \)  difference between the twisting angular momentum at takeoff and its average value in the period between takeoff and the peak
\( I_L \)  moment of inertia about the longitudinal axis
\( I_{L,avg} \)  average moment of inertia about the longitudinal axis in the interval between two frames
\( I_{L,avg} \)  average moment of inertia between takeoff and the peak
\( SD \)  standard deviation
\( \Delta t \)  time between takeoff and the peak
\( M \)  mean
\( X_B \)  mediolateral axis
\( Y_B \)  dorsiventral axis
\( Z_B \)  longitudinal principal axis
\( \alpha, \beta, \gamma \)  Cardan angles that define the rotation of the body from one frame to the next
\( \epsilon \)  angle between the resultant angular momentum vector and the horizontal plane
\( \eta_{TO,adj} \)  angle between the bar and a line normal to the vertical plane containing both hips at takeoff (adjusted and unadjusted)
\( \eta_{TO,unadj} \)  angle between the bar and a line normal to the vertical plane containing both hips at takeoff (unadjusted)
\( \theta \)  backward/forward tilt of the longitudinal axis at takeoff, relative to the vertical
\( \kappa \)  angle between the resultant angular momentum vector and the longitudinal axis at takeoff
\( \tau_{PK} \)  hip twist angle at the peak
\( \tau_{TO,adj} \)  hip twist angle at takeoff (adjusted and unadjusted)
\( \tau_{TO,unadj} \)  hip twist angle at takeoff (unadjusted)
\( \Delta \tau_{adj} \)  cumulative twist rotation between takeoff and the peak (adjusted and unadjusted)
\( \Delta \tau_{unadj} \)  cumulative twist rotation between takeoff and the peak (unadjusted)
\( \Delta \tau_{c,adj} \)  twist rotation produced by cutting (adjusted and unadjusted)
\( \Delta \tau_{c,unadj} \)  twist rotation produced by cutting (unadjusted)
\( \Delta \tau_H \)  twist rotation produced by the twisting component of angular momentum between takeoff and the peak
\( \phi \)  angle between the longitudinal axis at takeoff and the plane normal to the resultant angular momentum vector
\( \omega_L \)  angular velocity of a body segment about its own longitudinal axis
\( \omega_r \)  angular velocity of twisting of the body produced by the angular momentum
Figure 1 — Sequences of two high jumps. The graphics show views perpendicular to the plane of the bar and the standards, looking from the run-up apron toward the landing pit, and they progress from right to left. Both athletes took off from the left foot. (a) Normal jump; (b) undertwisted jump, with the right hip lower than the left hip at the peak of the jump.
Let us focus on the twist rotation. Motions of body segments relative to each other can make the longitudinal axis deviate from its original conical path and move it closer to (or farther from) the angular momentum vector (Dyson, 1970; Frohlich, 1980). Such changes in the angle between the longitudinal axis and the angular momentum vector will change the magnitude of $\mathbf{H}_T$. In turn, this will affect the angular velocity of the twist rotation, determined by the equation $\mathbf{\omega}_T = \frac{\mathbf{H}_T}{I_z}$, where $I_z$ is the moment of inertia about the longitudinal axis.

Changes in the body configuration can also alter the value of $I_z$, the other factor that governs the value of $\mathbf{\omega}_T$. This is a second mechanism through which changes in segment positions can affect the angular velocity of the twist rotation (Dyson, 1970; Frohlich, 1980).

An airborne person can rotate one part of the body in one direction to make other parts rotate in the opposite direction. These action-and-reaction rotations can produce a net rotation of the whole body without angular momentum, as demonstrated by the classical experiment where a cat dropped upside down manages to land on its feet (Guyou, 1894; Lévy, 1894; Marey, 1894; McDonald, 1960) and also by experiments with humans.
High jumpers may superimpose action-and-reaction twist rotations on the general twist rotation of the whole body. This is a third mechanism that may affect the twist rotation of the hips. (For brevity, all actions and reactions in the twist rotation will be referred to as “catting” in this paper.)

To understand the mechanisms that sometimes produce a tilted position at the peak of the jump with one hip lower than the other, we first need to know how the twist rotation is produced in normal high jumps. In theory, it can be produced through the twisting component of angular momentum, through catting, or through both. Given the known general direction of the angular momentum vector and the fact that the principal longitudinal axis is roughly vertical at the end of the takeoff (Figure 2c), we can infer that angular momentum produces an initial angular velocity of twisting at takeoff. However, the changes in the angle between the longitudinal axis and the angular momentum vector after takeoff are not known. Therefore, we do not know if angular momentum continues to contribute to twist rotation until the peak of the jump. It is also not known what contribution (if any) catting makes to the twist rotation.

The main goal of this paper was to find out whether angular momentum, catting, or a combination of the two is responsible for the twist rotation that occurs in high jumping between takeoff and the peak of the jump.

**Methods**

The jumps used in the study were chosen from a database gathered for previous research and service work. The database consisted of the three-dimensional (3D) coordinates of body landmarks in jumps by over 100 elite male and female high jumpers filmed or videotaped in competitions. Female high jumpers generate less vertical velocity than males and thus have less time to complete their rotations. Since this could lead to different strategies for the generation of twist rotation, men and women were studied separately. Ten normally twisted high jumps were selected for analysis: five trials by men and five by women. The protocol for the use of human subjects was reviewed and approved by the Indiana University Committee for the Protection of Human Subjects.

The jumps were recorded with two 16 mm motion-picture cameras or with two video cameras (50 Hz). Twenty-one body landmarks were digitized from the film or video images in every third frame of the two cameras. The time between digitized frames was considered sufficiently short to describe adequately the athletes’ motions during the airborne phase of the jump, when movements are much slower than in the run-up or takeoff.

The Nonlinear Transformation method (Dapena, 1985; Dapena, Harman, & Miller, 1982) was used to compute 3D coordinates in the early competitions; the Direct Linear Transformation method (Abdel-Aziz & Karara, 1971; Walton, 1981) was used for later competitions. (For more details, see Dapena, McDonald, & Cappaert, 1990.)

The files in the original database contained unsmoothed 3D coordinates of the body landmarks for instants spaced at 0.06 s intervals. However, for the present study it was more convenient to use data spaced at 0.02 s intervals. This was achieved through curvilinear interpolation using unsmoothed quintic spline curves (Wood & Jennings, 1979).

No trunk landmarks were digitized between hips and suprasternal. However, the fact that the trunk tends to arch backward when the thighs are hyperextended at the hip and forward when they are flexed (Dapena, 1993) was used to estimate the 3D positions of omphalion and xiphion, landmarks proposed by Zatsiorsky’s group for the separation of the trunk into three parts (Zatsiorsky & Seluyanov, 1983; Zatsiorsky, Seluyanov, & Chugunova, 1990a, 1990b). The average flexion–extension angle of the two thighs relative to the straight line joining the midhip and the suprasternal was used to estimate for
each instant a function that defined the curved midline of the trunk (Dapena, 1993). The coordinates of omphalion and xiphion were estimated from this function and from the lengths given by Zatsiorsky's group for the three trunk subsegments, with adjustments by de Leva (1996). Instantaneous 3D location and velocity values were computed for the 23 body landmarks from smoothed quintic spline functions fitted to the raw location values.

The body was modeled as a 16-segment mechanical system (Zatsiorsky & Seluyanov, 1983; Zatsiorsky et al., 1990a, 1990b; adjusted by de Leva, 1996). The 3D coordinates of the segmental and whole-body centers of mass were computed from body landmark coordinates (Dapena, 1978). Angular momentum was computed using the method described by Dapena (1978), with two alterations. One alteration was the use of instantaneous landmark velocities as input, instead of average velocities between frames. This made the method cleaner, although it probably did not have much effect on the results. The second alteration involved the calculation of each segmental angular velocity about its own longitudinal axis ($\omega_L$). In the original method, $\omega_L$ was assumed to be zero for all nontrunk segments. In the altered method, the $\omega_L$ vector was estimated for each arm segment and for the head as the projection of the angular velocity vector of the upper trunk on the segmental longitudinal axis, and for each leg segment as the projection of the angular velocity vector of the lower trunk on the segmental longitudinal axis.

Without air resistance, an airborne body has constant angular momentum about its own CM. However, errors in digitizing and in the segmental parameters produce fluctuations in the computed angular momentum, which could lead to misinterpretation of results. To avoid this risk, it was decided to use jumps in which the computed angular momentum was constant during the airborne phase. For this, the airborne phases of the selected trials were subjected to a simulation procedure. This was done following the method developed by Dapena (1981), modified for use with the new 3-segment trunk model.

The method involves two stages. In the first stage, the body landmark coordinates from an actual jump and the segment inertial parameters are used to compute the initial orientation (at takeoff) of a reference frame linked to the trunk, the successive orientations of the segments relative to the trunk, anthropometric lengths, the CM path, and the average angular momentum in the air. In the normal use of the method, part of the output from the first stage is then altered and used as input for the second stage, which generates an altered simulated airborne phase. The output from the second stage is a file with the body landmark coordinates of the simulated jump.

In the study described here, the output from the first stage was not altered before its use as input for the second stage. This produced a simulated jump almost identical to the original but with constant angular momentum. The simulated jump was used in the study, thus ensuring that the mechanical parameters of the jumps used in the study complied with Newton’s laws of mechanics, and that the cause–effect mechanisms found were real and were not artifacts stemming from fluctuations (errors) in the angular momentum. The orientations of the trunk at the peak of the jump in the original and simulated jumps were compared to ensure that the differences between them were small.

The principal moments of inertia and the orientations of the principal axes were computed with the method proposed by Hinrichs (1978). The magnitudes of the largest and intermediate principal moments of inertia were similar during the period between takeoff and the peak of the jump, and this led to large fluctuations in the orientations of the corresponding axes during that period. The fluctuations did not correspond with the intuitive concept of twist rotation of the body and therefore rendered these principal axes unsuitable for use in the present study. However, the principal axis associated with the smallest moment of inertia retained during the entire period a clear identity with the concept of
the "longitudinal" axis of the body and therefore was quite useful for describing somersault rotation.

The longitudinal principal axis of the body and the locations of the hips were used to establish a reference frame defined by axes $X_b$, $Y_b$, and $Z_b$ which described the general 3D orientation of the body. $Z_b$ coincided with the longitudinal principal axis and pointed from the CM toward the head; $Y_b$ was dorsiventral, the cross-product of $Z_b$ with a vector pointing from the left hip toward the right hip; and $X_b$ was mediolateral, the cross-product of $Y_b$ with $Z_b$.

The angle between the resultant angular momentum vector and the horizontal plane ($\epsilon$) was computed from the 3D components of the vector. The angle $\kappa$ between the resultant angular momentum vector and the vector aligned with the longitudinal axis ($Z_b$) at takeoff was computed from their 3D components. (From here on, takeoff will refer to the first frame in which the body was airborne.) The angle between the longitudinal axis at takeoff and the plane normal to the angular momentum vector was computed using the equation $\phi = 90^\circ - \kappa$. (Positive values of $\phi$ represented forward lean relative to the plane.) The backward/forward tilt of the longitudinal axis at takeoff relative to the vertical was computed as the angle $\theta$ between the longitudinal axis and the vertical plane normal to the plane defined by the longitudinal axis and the angular momentum vector. (Negative $\theta$ angles implied backward lean.)

High jumpers start twisting counterclockwise already during the takeoff phase, and at the end of takeoff the hips are facing somewhat away from the pit. The angle between the bar and a line normal to the vertical plane containing both hips was computed for the instant of takeoff. Positive values were assigned to this initial hip orientation angle $\eta_{TO}$ when the hips faced away (counterclockwise) from the landing pit.

The twist orientation of the hips at the peak of the jump is ultimately what decides whether an athlete twisted enough, so it is an important parameter (see below). The orientation of the hips at takeoff is less informative. The hips of one subject may be less rotated, but the legs or arms more rotated, than those of another subject. Although the hips make the first subject appear to be behind in the twist rotation at takeoff, action and reaction could easily be used to advance the twist rotation of the hips through reverse rotations of the legs or arms. What matters at takeoff is the overall orientation of the whole body, rather than the hip orientation.

Angle $\eta_{TO}$ as defined above describes well the hip orientation at takeoff but has only limited validity for comparing overall twist orientation of the whole body in different subjects. To improve the validity of the comparisons, an adjusted initial hip orientation angle was computed for each jump, based on a standardized body configuration. For this, an average body configuration at takeoff was computed from a sample of jumpers taken from the high jump database (Dapena et al., 1990). Then, for each of the subjects in the present study a brief computer simulation was carried out in which the body configuration was changed from the actual configuration at takeoff to the standardized configuration, with angular momentum kept at zero. The simulation produced new orientations for all body segments relative to the ground. Since the simulation involved no angular momentum, the new (hypothetical) body position was considered equivalent to the original body position with regard to rotation, and it had the advantage of being associated with the same standardized body configuration in all subjects. Therefore, the new body position was used to compute an adjusted value for the hip orientation angle at takeoff ($\eta_{TNADJ}$). This adjusted value was subsequently used for comparisons among jumps. (The unadjusted hip orientation angle at takeoff $\eta_{TO}$ was relabeled $\eta_{TNUNADJ}$.)

The projection of a vertical vector on the plane normal to the longitudinal axis at the peak of the jump defined the neutral ("face-up") twist orientation. The hip twist angle at
the peak \( (\tau_{PK}) \) was the angle between axis \( Y_B \) and the neutral twist orientation vector. (The value \( \tau = 90^\circ \) was assigned to the neutral twist orientation at the peak; in untwisted jumps, \( \tau_{PK} < 90^\circ \).)

Axes \( X_B, Y_B, \) and \( Z_B \) defined the body orientation (see above). Rotation from one frame to the next was defined by the Cardan angles \((\alpha, \beta, \) and \( \gamma)\) of the three successive rotations about the \( X_B, Y_B, \) and \( Z_B \) axes of the first frame that turned the axes into their orientations in the next frame; the second and third rotations were about axes displaced by the previous rotation(s). The third angle \((\gamma)\) indicated the twist rotation between the frames. The cumulative twist rotation between takeoff and the peak was computed as the sum of the \( \gamma \) angles in all the intervals. Two values were obtained: \( \Delta \tau_{\text{UNADJ}} \), which started from the actual position of the body at takeoff, and \( \Delta \tau_{\text{ADJ}} \), which started from the position associated with the standardized body configuration at takeoff (see above). \( \Delta \tau_{\text{ADJ}} \) was considered more appropriate for comparisons among jumps.

The cumulative twist rotation was subtracted from the hip twist angle at the peak to compute the hip twist angle at takeoff. The difference between the hip twist angle at takeoff and \( 90^\circ \) indicated the angle through which the athlete would need to twist in the air in order to be in the neutral twist orientation at the peak \((\tau_{PK} = 90^\circ)\). Again, two values were obtained: an unadjusted value \( \tau_{\text{TO UNADJ}} \) and an adjusted value \( \tau_{\text{TO ADJ}} \). The latter was considered more appropriate for comparisons among jumps.

The procedure for calculating cumulative twist rotation associated with twisting angular momentum started with the projection of the angular momentum vector \( \mathbf{H} \) in each frame on the longitudinal axis of the body \( Z_B \). This yielded the twisting component of angular momentum \( H_T \). A positive sign was given to \( H_T \) (a scalar with the magnitude of \( H_T \)) if the vector pointed toward the head. The angular velocity of twisting associated with \( H_T \) in each interval between two consecutive frames was calculated as \( \omega_T = \frac{H_{T_{\text{avg}}}}{I_{L_{\text{avg}}}} \), where \( H_{T_{\text{avg}}} \) and \( I_{L_{\text{avg}}} \) were the average values of \( H_T \) and of the moment of inertia about the longitudinal axis \( I_L \) in the two frames. The product of \( \omega_T \) and the interval duration \((0.02 \text{ s})\) yielded the amount of twist rotation associated with \( H_T \) during the interval. The amount of twist rotation produced by the twisting component of angular momentum between takeoff and the peak \( (\Delta \tau_H) \) was calculated as the sum of the twist rotations associated with \( H_T \) in the intervals between takeoff and the peak.

The difference between the total cumulative twist rotation of the hips and the cumulative twist rotation of the hips associated with \( H_T \) was attributed to action and reaction (cating). Two values were obtained for the twist rotation attributed to cating: an unadjusted value, \( \Delta \tau_{\text{CUNADJ}} = \Delta \tau_{\text{UNADJ}} - \Delta \tau_H \), and an adjusted one, \( \Delta \tau_{\text{ADJ}} = \Delta \tau_{\text{ADJ}} - \Delta \tau_H \).

A test was designed to check the validity of the method used to separate the twist rotation into fractions due to angular momentum and to cating. For this test, special simulated jumps were generated with two of the athletes in the sample. In these simulated jumps, the conditions at takeoff were left the same as in the original jumps, but the body was kept “frozen” in the takeoff configuration during the entire airborne phase. Subsequently, these two rigid-configuration jumps were analyzed with the computer programs that were used to analyze the normal high jumps. If the method for separating the twist rotation into fractions was valid, in each of these jumps the programs should have attributed all the twist rotation to the twisting component of angular momentum, and none to cating.

**Results and Discussion**

The personal records of the subjects and the maximum heights that they cleared in the analyzed competitions (Table 1) denoted their status as elite high jumpers. For comparison, the heights needed to qualify for the finals of the men’s and women’s high jump at the
1992 Olympic Games were 2.26 m and 1.92 m, respectively (Gordon, 1992; Saylors, 1992). The standing heights and masses of the subjects (Table 1) were similar to those of the top 10 high jumpers in the world in 1995 (men: 1.92 ± 0.05 m, 75 ± 4 kg; women: 1.79 ± 0.02 m, 61 ± 3 kg) (Quercetani et al., 1995a, 1995b).

The time from takeoff to the peak of the jump was 0.43 ± 0.02 s for the men and 0.37 ± 0.02 s for the women. At the peak, the root mean square difference between orientation of the upper trunk in the simulated jumps and in the original jumps was 5° (men) and 3° (women) for rotation about a transverse axis of the upper trunk (somersault error), and 3° (men) and 3° (women) for rotation about the longitudinal axis of the upper trunk (twist error). When the directions of the differences were accounted for, the difference in the twist orientation was –2 ± 2° (men), and 0 ± 3° (women); negative values implied relative undertwist of the simulated jump. The small sizes of these differences indicated a great similarity between the original jumps and the simulated jumps and implied that the benefit of using simulated jumps (constant angular momentum) was not obtained at the expense of excessive changes in the rotations of the athletes.

Analyses of the two rigid-configuration artificial jumps attributed 62.7° and 74.1°, respectively, to the twist rotation produced by the twisting component of angular momentum, and –0.4° and –0.6°, respectively, to the twist rotation produced by cattig. Given the athlete’s rigid configuration during the airborne phases of these simulated jumps, the program should have attributed zero twist rotation to cattig in both cases. The error of about half a degree was probably caused mainly by inaccuracies due to the use of discrete 0.02 s time intervals for the calculations. This error was quite small, and the test supported the validity of the algorithm to separate twist rotation into fractions due to twisting angular momentum and to cattig.

The women’s hip orientation relative to the bar at takeoff (π\textsubscript{TOADJ}) was similar to that of the men (Table 2). The average hip twist angle of the women at takeoff (τ\textsubscript{TOADJ}) was somewhat larger than that of the men, although there was quite a bit of overlap between the two groups.

It may seem odd that at takeoff the women were, on the average, closer to the face-up position than the men (larger τ\textsubscript{TOADJ} angles) when the initial hip orientation angle (π\textsubscript{TOADJ}) was about the same for both groups. However, this was not an inconsistency: The amount of twist rotation needed to reach the face-up position at the peak of the jump depends not only on the hip orientation at takeoff but also on the particular 3D path followed by the longitudinal axis in its somersault rotation between takeoff and the peak.

The women went through a slightly smaller twist rotation between takeoff and the peak of the jump (Δτ\textsubscript{ADJ}) than the men, although there was again overlap between them.

Table 1 Characteristics of the Subjects

<table>
<thead>
<tr>
<th></th>
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<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Standing height (m)</td>
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<tr>
<td>Mass (kg)</td>
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<tr>
<td>Personal record (m)</td>
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<td>Height jumped at the analyzed competition (m)</td>
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Table 2  Twist Orientations and Rotations of the Hips (in Degrees)

<table>
<thead>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>η_{TO(ADD)}</td>
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<td>τ_{TO(ADD)}</td>
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<td>7</td>
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<td>Δτ_{UnADD}</td>
<td>46</td>
<td>11</td>
<td>52</td>
<td>13</td>
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<tr>
<td>Δτ_{C(UnADD)}</td>
<td>13</td>
<td>14</td>
<td>37</td>
<td>13</td>
</tr>
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</table>

Note. η_{TO(ADD)} = adjusted hip orientation at takeoff, relative to the bar; τ_{TO(ADD)} = adjusted hip twist angle at takeoff; Δτ_{ADD} = adjusted hip rotation between takeoff and peak; Δτ_{H} = hip rotation associated with twisting component of angular momentum; Δτ_{C(ADD)} = adjusted hip rotation associated with cattting; τ_{PK} = twist angle at peak. The unadjusted values of η_{HO}, τ_{HO}, Δτ, and Δτ_{C} are given in the last four rows.

(Table 2). The men’s larger twist rotation compensated for their smaller initial twist orientation, and the twist angle at the peak (τ_{PK}) was essentially the same for both groups. Of course, this had to be so, since a twist angle near 90° was the selection criterion for inclusion in the two samples.

The women achieved more twist rotation through cattting than the men (men: Δτ_{C(ADD)} = 22 ± 6°; women: Δτ_{C(ADD)} = 34 ± 5°), while the men twisted more through the twisting component of angular momentum (men: Δτ_{H} = 33 ± 7°; women: Δτ_{H} = 14 ± 8°). This was a marked difference between the two group samples, the reasons for which are not clear; perhaps this difference is related to greater flexibility in the women. But the most interesting finding was that the contribution of cattting to the twist rotation in the pooled sample of men and women was larger than that of the angular momentum (Δτ_{C(ADD)} = 28°; Δτ_{H} = 24°). Such a large relative contribution by cattting to the twist rotation was initially surprising. This was because the mechanics of airborne twisting has generally been studied in activities that require large amounts of twist. In those activities (such as fast twisting dives or gymnastics jumps), twist rotation is produced mainly through angular momentum, because the twist that can be generated through cattting is limited in comparison with the total amount required (Batterman, 1977; Yeadon, 1993). The results of the present paper indicate that in activities that require smaller amounts of twist rotation, such as high jumping, the contribution of cattting to the twist rotation can be as large as that of the angular momentum.

The data in Table 3 show that the smaller twist rotation achieved by the women through angular momentum was due mainly to their smaller average twisting angular momentum between takeoff and the peak (H_{I(AVG)}: ratio women/men = 0.51) and to lesser extents to their larger average moment of inertia (I_{C(AVG)}: ratio men/women = 0.77) and shorter time available (Δt: ratio women/men = 0.86).
Table 3  Factors That Affect Contribution of the Twisting Component of Angular Momentum to Twist Rotation

<table>
<thead>
<tr>
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<th>Women</th>
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<tbody>
<tr>
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<td>$M$</td>
<td>$SD$</td>
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<tr>
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<td>$\Delta H$ (s$^{-1}$, 10$^{-3}$)</td>
<td>-22.9</td>
<td>6.3</td>
</tr>
<tr>
<td>$I_{\text{LAVG}}$ (kg$\cdot$m$^2$/kg$\cdot$m$^2$ 10$^{-3}$)</td>
<td>12.4</td>
<td>1.0</td>
</tr>
<tr>
<td>$\Delta t$ (s)</td>
<td>0.43</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Note. $\varepsilon$ = angle between resultant angular momentum vector and horizontal plane; $\theta$ = angle of forward–backward tilt of longitudinal axis of the body at takeoff; $\phi$ = angle between plane normal to angular momentum vector and longitudinal axis at takeoff ($\phi = \varepsilon + \theta$); $H$ = magnitude of resultant angular momentum vector; $H_{\text{TANGENT}}$ = initial and $H_{\text{TAVG}}$ = average values of twisting component of angular momentum; $\Delta H$ = change from initial to average twisting component of angular momentum; $I_{\text{LAVG}}$ = average moment of inertia about longitudinal axis between takeoff and the peak; $\Delta t$ = time between takeoff and peak. To facilitate comparisons, all angular momentum and moment of inertia values were normalized: They were divided by the product of the mass and the square of the standing height of each subject.

Computer graphics showed that the women tended to keep their legs farther apart than the men, which is probably why their $I_{\text{LAVG}}$ values were larger. The reasons for this choice are not clear. Maybe the more spread-out positions of the legs reduced the moment of inertia about the mediolateral axis and helped to speed up the somersault. (Women have less time available for their rotations than men, so they may seek to increase angular velocity in order to achieve an adequate amount of somersault rotation in less time.) The somersault rotation was outside the scope of this study, so this question was not pursued further.

While the wider separation of the legs may have decreased the women’s moment of inertia about the mediolateral axis, it increased the moment of inertia about the longitudinal axis and thus contributed to the women’s smaller $\Delta \tau_{\text{H}}$ values. However, the more far-apart positions of the women’s legs may also have facilitated the generation of larger amounts of twist rotation through caging ($\Delta \tau_{\text{CADDY}}$), thus possibly compensating for the smaller $\Delta \tau_{\text{H}}$ values.

Figure 3 shows two typical jumps (a = men; b = women). Each image is seen from a direction normal to the plane defined by the resultant angular momentum vector and the longitudinal axis at that time. (The direction of the view changed with each successive image, keeping pace with the somersault of the longitudinal axis.) The single-headed and double-headed arrows represent the total angular momentum and the principal longitudi-
nal axis, respectively; the line is the plane normal to the angular momentum vector (see Figure 3c). After takeoff, the longitudinal principal axis tilted toward the plane in all subjects, reducing the twisting component of angular momentum. In the men, the longitudinal axis almost never reached the plane (Figure 3a). But the longitudinal axis did reach the plane in almost all the women, and actually crossed to the other side (Figure 3b); this reversed the sign of the twisting component of angular momentum, which made a negative contribution to the twist rotation from that time to the peak of the jump.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Sequences of typical jumps by (a) male and (b) female high jumpers, viewed at each instant from a direction perpendicular to the plane formed by the current instantaneous longitudinal axis of the athlete and the resultant angular momentum vector. Both sequences progress from right to left. (c) A subject at the instant of takeoff, with an explanation of the meanings of the arrows and lines. Between takeoff and the peak of the jump, the longitudinal axis tilted gradually toward the plane normal to the angular momentum vector and often went beyond that plane in the women.}
\end{figure}
The orientation of the principal longitudinal axis of the body was roughly similar to
that of the longitudinal axis of the trunk. The tilt of the longitudinal axis of the body
toward the plane normal to the angular momentum (a clockwise rotation in Figure 3)
seemed to be due mainly to the counterclockwise rotations implicit in the lowering of
the lead leg and the lifting of the trailing leg behind the body. Due to the decrease in the
twisting component of angular momentum \( H_t \) after takeoff, the average value of this
parameter between takeoff and the peak of the jump \( H_{T\text{avg}} \) was smaller than its initial
value \( H_{T\text{init}} \) in all subjects.

It was pointed out before that the women’s smaller \( \Delta \tau_H \) values in relation to those of
the men were due mainly to the women’s smaller average twisting angular momentum
between takeoff and the peak \( \text{men: } H_{T\text{avg}} = 15.8 \pm 3.7 \cdot 10^{-3} \text{ s}^{-1}; \text{women: } H_{T\text{avg}} = 8.0 \pm
5.0 \cdot 10^{-3} \text{ s}^{-1} \). The women already had less twisting angular momentum at takeoff \( \text{men: } H_{T\text{init}} = 38.7 \pm 5.3 \cdot 10^{-3} \text{ s}^{-1}; \text{women: } H_{T\text{init}} = 34.7 \pm 4.5 \cdot 10^{-3} \text{ s}^{-1} \), and then lost more of
it after takeoff \( \text{men: } \Delta H_t = -22.9 \pm 6.3 \cdot 10^{-3} \text{ s}^{-1}; \text{women: } \Delta H_t = -26.7 \pm 4.4 \cdot 10^{-3} \text{ s}^{-1} \).
The women’s smaller initial twisting angular momentum and greater losses of it
after takeoff contributed in roughly equal amounts to their smaller average twisting
angular momentum between takeoff and the peak \( H_{T\text{init}} \); difference women−men =
\(-4.0 \cdot 10^{-3} \text{ s}^{-1} \); \( \Delta H_t \); difference women−men = \(-3.8 \cdot 10^{-3} \text{ s}^{-1} \).

In summary, women’s twist orientations at takeoff and at the peak were roughly
similar to those of the men. Both groups achieved their twist rotations through a combina-
tion of the basic rotation (twisting component of angular momentum) and catting, but the
men tended to rely more on the basic rotation and the women on catting. The women’s
basic rotation was smaller than the men’s due to the women’s smaller initial twisting
angular momentum and greater losses of it after takeoff, larger average moment of inertia
about the longitudinal axis, and less time available between takeoff and the peak.

**Conclusions**

At takeoff, the longitudinal axis of a high jumper is at an oblique angle with respect to the
angular momentum vector, and this produces a counterclockwise twist rotation. However,
in the air the longitudinal axis tilts toward the plane normal to the angular momentum
vector, which reduces the speed of the twist rotation produced by the angular momentum.
In many cases the longitudinal axis crosses the plane, and the twisting component of
angular momentum (and its contribution to the twist rotation) reverse direction before the
peak of the jump.

The most important finding of the study was the major role played by catting in the
generation of the twist rotation: In the pooled 10-subject sample, catting was responsible
for more than half of the total twist rotation. This important role of catting introduces the
possibility that defects in its execution might be a factor in the twist rotation problems of
some high jumpers. This implies that in diagnosing problems in a high jumper’s twist
rotation, researchers and coaches should consider not only the angular momentum of the
athlete and how the longitudinal axis changes its tilt relative to the angular momentum
vector, but also the amount of twist rotation that is produced through catting.

The results of the study also raised new questions: How does the 3D path followed
by the longitudinal axis in its somersault rotation between takeoff and the peak of the
jump affect the amount of cumulative twist rotation necessary to achieve a face-up orient-
ation at the peak of the jump? How is catting produced? Why did the women generally
make more use of catting, while the men made more use of angular momentum? These
questions should be addressed by further research. It will also be important to compare
undertwisted jumps with normally-twisted jumps, to try to find the most usual causes for the differences between them.

References


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