Optimum Angles of Projection in the Throws and Jumps
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Introduction

Many athletics coaching manuals include a section on the biomechanics of the throwing events. A brief summary of the 'science of projectile motion' is given, followed by the advice that the athlete should strive for maximum projection speed and should project the implement at about 45°. The advice on maximum speed seems correct, but what about the optimum projection angle? Measurements of competitors at the Olympic Games and World Championships often show projection angles that are substantially lower than 45°. For example, shot-putters have projection angles of from 26° to 42°, with an average value of about 37°. Why the discrepancy between theory and practice? Have the athletes got it wrong and could their performances be improved by raising their projection angles to match the theoretical optimum angle?

In this article I argue that it is the athletes who have got it right and that it is the theory that is not quite correct. The error in the conventional theory is that it does not take into account the fact that the projection speed of the implement decreases when you throw with a higher projection angle. I will first consider the optimum projection angle in shot-putting, and then consider how the principles also relate to the other throwing events (hammer, javelin and discus) and to the jumping events (long jump).

Projectile motion

Centuries of warfare have resulted in a thorough understanding of ballistics (the motion of projectiles fired from cannons, artillery and the like). This same knowledge has also been applied to the analysis of sports projectiles. Most people have heard that if you wish to maximize the range of a projectile you must launch it at an angle of 45°. However, this is only true if the launch and landing are on the same level. (For example, a cannon fired on a level battlefield.) Things get a little more complicated when the projectile is launched from above ground level, as in the shot put (see Figure 1).

Figure 1

Shot putting factors that affect the horizontal range of the shot
Equation 1 will not mean much to most athletics coaches, so I have made it easier to understand by showing it graphically in Figure 2. The range of the shot is plotted against the projection angle. Curves are shown for a selection of projection speeds, from a very modest speed of 8 metres per second (m/s) through to a speed slightly greater than that produced by an elite shot-putter (15 m/s). The calculations are with \( h = 2.10 \)m, which is the usual height difference between the launch and landing for adult male shot-putters. Please see the Appendix if you are not familiar with the units of speed (metres per second) used here.

Equation 1

\[
d_{\text{flight}} = \frac{v^2 \sin 2\theta}{2g} \left[ 1 + \left( 1 + \frac{2gh}{v^2 \sin^2 \theta} \right)^{1/2} \right]
\]

Figure 2

Consider a shot projected at constant speed, say 13 m/s. (Find this curve on Figure 2.) As the projection angle is raised, the athlete is able to throw farther and farther until a maximum range of 19 m is achieved at an optimum projection angle of about 42°. For higher projection angles the distance
achieved decreases again, eventually becoming zero when the shot is projected vertically up (projection angle = 90°). Note also that the maximum distance achieved by the shot-puter increases as the projection speed is increased. (Compare the curve for 10 m/s with that for 15 m/s.) Hence the common advice to shot-putters; strive to achieve the highest possible projection speed, and throw at an angle of just under 45°.

A question for the coach

Now consider the following scenario. You are a coach of an experienced shot-puter. You are an educated coach, well aware of the science of ballistics and its application to sports projectiles. You have measured your athlete's projection angles from a video of their competition throws. The projection angles on all the best throws are around 31°-33°. The preceding discussion suggests that the optimum projection angle in shot-putting is about 42°. Should you increase your athlete's projection angle? Will your athlete throw farther if they project the shot at the theoretical optimum angle of 42°?

A revised optimum projection angle

The answer is that your athlete will not improve if they raise their projection angle. Your athlete is probably already throwing at their optimum projection angle. Any changes to their projection angle will result in worse performances rather than better. The athlete and coach have got it right. It is the theory that is not quite correct.

The preceding discussion on the optimum projection angle in shot-putting has neglected the 'human element'. The calculations did not include the fact that an athlete cannot throw with the same speed at all projection angles. A shot-puter who has a maximum projection speed of 13 m/s at a projection angle of 0° will not be able to produce 13 m/s at angles of 30°, 60° and 90°. Figure 3 shows the relation between the projection speed and projection angle for a male college shot-puter. Note that the projection speed steadily decreases as the athlete tries to throw with a higher and higher projection angle.

Figure 3
The decrease in projection speed with increasing projection angle is a result of two factors.

1. When throwing with a high projection angle, the shot-putter must expend a greater effort during the delivery phase to overcome the weight of the shot, and so less effort is available to accelerate the shot (i.e. produce projection speed).

2. The structure of the human body favours the production of putting force in the horizontal direction more than in the vertical direction. Considering just upper body strength, most athletes can lift more weight in a bench press exercise than in a shoulder press exercise.

The correct optimum projection angle for our college shot-putter is obtained by combining Figure 3 with the equation for the range of a projectile (Equation 1). The equation for the curve in Figure 3 is substituted into Equation 1, and the range of the shot is plotted against the projection angle. The result is shown in Figure 4. As the projection angle is raised, the projection speed that the athlete is able to generate decreases, and so the range achieved (solid line) is lower than if the athlete were able to maintain a constant projection speed (dashed lines). The optimum projection angle for this athlete is not 42°, but about 32°. Please don't be worried if you cannot see exactly how the curve in Figure 4 was obtained; it requires some mathematical manipulation. The interested reader is referred to a more detailed scientific paper (Linthorne, 2001).

Figure 4
Unfortunately, all athletes do not have speed versus angle curves that have exactly the same shape as that shown in Figure 3. Each athlete has a unique speed versus angle curve that depends on their strength, stature and throwing technique. This means that the optimum projection angle is different for each athlete. An athlete whose projection speed decreases much less rapidly than shown in Figure 3 will have a higher optimum projection angle (for example, 37°), and an athlete whose projection speed decreases more rapidly will have a lower optimum projection angle (for example, 28°). For elite shot-putters the optimum projection angle usually lies between 30° and 40°.

**Projection speed is more important than projection angle**

The curve in Figure 4 shows that projecting the shot at the optimum angle is not crucial to successful shot-putting. Relatively large errors in projection angle are tolerable. Examine closely the region of the solid curve near to the athlete’s optimum projection angle (32°). If this athlete produces a projection angle that deviates by 3° from the optimum angle, his performance will be only 10 cm less than the maximum possible distance (16.6 m). Therefore, getting the projection angle right is not really that important. It is much more important for the athlete to generate a high projection speed than it is to project the shot at the optimum angle.

**Increasing the force on the shot improves performance**

If an athlete is already throwing at close to their optimum projection angle, performance can only be improved by increasing their projection speed. There are two ways of increasing projection speed. The most obvious way is through strength training to increase the force the athlete can exert on the shot during the delivery phase. Greater force results in a higher projection speed and hence a longer throw. Increases in projection speed may also be achieved through technique training. Here the athlete aims to exert more force on the shot by using their current level of strength more effectively.

Initially, the athlete has a maximum performance of 16.6 m. This is labelled the 100% strength level. Following a period of strength training, the athlete exerts a greater force on the shot and so produces a greater projection speed. The athlete throws farther, but the athlete’s optimum projection angle does not change much. An increase in strength to just over 140% of the initial level is required for this athlete to produce a World Record throw of 23.2 m. Technique training will also improve performance, but changes in technique may also alter the shape of the athlete’s projection speed versus projection angle curve (Figure 3). The athlete’s optimum projection angle may then be considerably different.

**Other throwing and jumping events**

The principles presented here also apply to the other throwing events. In all the throwing events the speed the athlete can generate decreases as the projection angle is increased. The optimum projection angle is therefore always less than 45°, and it is different for each event and for each athlete. In the hammer throw, the projection speed an athlete can produce decreases only slightly with increasing projection angle. This means that the optimum projection angle is relatively high, usually about 40°. Javelin throwers, on the other hand, show a relatively steep decrease in projection speed as the projection angle is increased, and so the optimum projection angle is relatively low. The javelin and discus throws differ from the shot put and hammer throw in that the implement is not in free flight. Aerodynamic lift and drag play a large part in determining the trajectory of the implement, and so the calculation of the optimum release angle involves very complex equations. (Even more complex than Equation 1 !) Typical optimum projection angles are about 33° in the javelin throw (Bartonietz, 2000), and about 35° in the discus throw.
The throwing events are not the only athletics events involving projectiles. In the long jump the projectile is the human body and the athlete wishes to maximize the range of this projectile. Again the optimum take-off angle is not 45°. The take-off speed a jumper can generate decreases rapidly as the take-off angle is increased. An elite male long jumper can produce a take-off speed of about 10.5 m/s in a 'run-through (i.e. a take-off angle of 0°), but can only reach about 3.5 m/s in a vertical jump (i.e. a take-off angle of 90°). This rapid decrease in take-off speed means that the optimum take-off angle is very much less than 45°, and most elite long jumpers have an optimum take-off angle of between 18° and 25° (Linthorne et al., 2001). As in the shot put, the optimum take-off angle is different for each athlete because of individual differences in the rate of decrease in take-off speed with increasing take-off angle.

**Determining the optimum projection angle**

Although it is possible to calculate the optimum projection angle for any athlete, it is not always easy to do so in practice. The calculation requires knowledge of how the athlete's projection speed changes with projection angle. You need to take very careful measurements using a high-speed video camera of many throws (or jumps) by the athlete over a wide range of projection angles. Such measurements usually require the services of a sports scientist, and the process is time-consuming and expensive. For the coach without the necessary resources, a process of trial-and-error at the training track is a good method of identifying your athlete's optimum projection angle. Most athletes will usually 'home-in' on their optimum projection angle relatively quickly with the aid of a tape measure (to measure their performance) and with the guidance of a coach (to help the athlete maintain good technique while experimenting with different projection angles). A high-speed video analysis is beneficial only to the very best athletes. Elite athletes are closer to their ultimate potential, so expending the time and resources to obtain an extra small improvement in performance may be justified.

**Conclusion**

In the throwing and jumping events, the optimum projection angle is usually considerably less than 45° because the speed an athlete can produce decreases as the projection angle is raised. Unfortunately for the coach, each athlete has a unique curve of projection speed versus projection angle. This means that the optimum projection angle is different for every athlete. However, it is not necessary for the coach to spend a great deal of effort in getting their athletes to throw or jump at the optimum projection angle. Most athletes find their optimum projection angle relatively quickly through trial-and-error, and achieving a high projection speed is much more important than throwing or jumping at the optimum angle.

**References**


**Appendix**

Speed is a measure of the rate that a moving object covers distance. If you measure distance in metres (m) and time in seconds (s), then the units of measurement of speed will be metres per second (m/s). You can measure distance in other units, for example, kilometres (km), miles (mi), or feet (ft). Likewise you can measure time in minutes (min), hours (hr), or days (d). Therefore you can have other units for speed, for example, kilometres per hour (km/hr), miles per minute (mi/min), or feet per day (ft/d).

For those readers not familiar with speed measurements in metres per second (m/s), here are some conversions:

- $1 \text{ m/s} = 3.6 \text{ km/hr}$
- $1 \text{ m/s} = 2.2 \text{ mi/hr}$

**Examples:**

- $13 \text{ m/s} = 13 \times 3.6 \text{ km/hr} = 47 \text{ km/hr}$
- $13 \text{ m/s} = 13 \times 2.2 \text{ mi/hr} = 29 \text{ mi/hr}$